A Novel Robust Nonlinear Motion Controller with Disturbance Observer

Zi-Jiang Yang, Hiroshi Tsubakihara, Shunshoku Kanae, Kiyoshi Wada and Chun-Yi Su

Abstract—In this paper, a novel robust nonlinear motion controller with disturbance observer (DOB) for positioning control of a nonlinear single-input-single-output (SISO) mechanical system is proposed. The controller is designed in a backstepping manner. At the first step, a PI controller is designed to stabilize the position error. Then at the second step, a novel robust nonlinear velocity controller with DOB is designed to stabilize the velocity error. By using some elegant nonlinear damping terms, the input-to-state stability (ISS) property of the overall nonlinear control system is proved, which leads to a major contribution of construction of a theoretically guaranteed robust nonlinear controller with DOB for the first time in the literature. The performance of the proposed controller is verified through application to a magnetic levitation system. Comparative studies with an adaptive robust nonlinear controller are also carried out. It is shown that the proposed novel controller while being simple is superior over the adaptive robust nonlinear controller for the experimental setup under study.

I. INTRODUCTION

The DOB based motion controllers have been widely accepted in the industrial side, due to their simplicity and transparency of design, and excellent disturbance compensation ability. The key point of a DOB is to pass the external disturbances and model mismatch lumped as an error term of the motion equation through a low-pass filter \( Q(s) \) and then to compensate the external disturbance and model mismatch by the output of the low-pass filter. The filter’s output is viewed as the estimate of the lumped disturbance. This results in an inner-loop around the controlled plant such that the inner-loop approximates a simple nominal plant model at low-frequencies. And hence a simple controller can be designed for the approximated nominal model.

So far, many papers have been published on the DOB based motion controllers [1–3], [5], [6]. A major problem of the DOB based controller is that the inner-loop approximates a simple nominal plant model only at low-frequencies, therefore the disturbances and model mismatch at high-frequencies may degenerate the control performance and even destroy the closed-loop stability. It was pointed out in [8] theoretically and experimentally that the DOB based controllers may not be robust to large model mismatch.

Many efforts have been paid to handle this problem. In [6], a robust two-degree-of-freedom control system including a DOB was proposed. In [5], the DOB based controller was designed using the linear \( H_{\infty} \) control theory. And in [3], a sliding mode controller was introduced to counteract the disturbance estimation error under the assumption that the linear nominal model is stabilized by an inner two-degree-of-freedom controller.

Despite so many works mentioned above, it should be commented here that the DOB based motion controllers are usually designed according to the linear control theory, even when the actual controlled plant may be strongly nonlinear. Unfortunately, the rigorous stability of these controllers for nonlinear systems has not been well studied in the literature. An exception can be found in [1], where the equivalence between a passivity based robot controller and a DOB based robot controller was investigated. And it was proved that the disturbed system of the velocity-loop is \( L_p \) input/output stable with respect to the pair of lumped disturbance (as the input) and velocity-loop error (as the output). However, no active efforts were paid to counteract the disturbance estimation error. Also, it is still unclear if the lumped disturbance itself is bounded in the case where some internal signals associated with large model mismatch are included in the lumped disturbance.

In this paper, a novel and theoretically guaranteed robust nonlinear motion controller with DOB for positioning control of a nonlinear SISO mechanical system is proposed. The controller is designed in a backstepping manner. At the first step, a PI controller is designed to stabilize the position error. Then at the second step, a novel robust nonlinear velocity controller with DOB is designed to stabilize the velocity error. The key point is to employ some elegant nonlinear damping terms to guarantee the ISS of the overall nonlinear control system. The complicated looking controller can be explained as modifications of the conventional PI motion controller with minor-loop, by adding the feedforward term, nonlinear damping term and DOB term to it. Therefore it is believed that the proposed controller may gain wide acceptance of the engineers owing to its simplicity of structure and transparency of design. The performance of the proposed controller is verified through application to a magnetic levitation system. Comparative studies with an adaptive robust nonlinear controller are also carried out. It is shown that the proposed novel controller while being simple is superior over the adaptive robust nonlinear controller for the experimental setup under study.
II. STATEMENT OF THE PROBLEM

Consider the following SISO nonlinear mechanical system:
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= F(x) + d(x, t) + G(x)u
\end{align*}
\]  
(1)

where, \( x = [x_1, x_2]^T \), \( x_1 \) and \( x_2 \) are the position and velocity respectively, \( u \) is the control input; \( G(x) \) and \( F(x) \) are modelable nonlinear functions with known nominal functions; and \( d(x, t) \) is the completely unknown term composed of unmodelled nonlinearities and disturbances.

Denoting the nominal nonlinearities based on priori knowledge as \( F_0(x) \) and \( G(x) \), we can model \( F(x) \) and \( G(x) \) as
\[
F(x) = F_0(x) + \Delta_F(x), \quad G(x) = G_0(x) + \Delta_G(x)
\]  
(2)

where \( \Delta_F(x) \) and \( \Delta_G(x) \) denote the modelling errors.

Some assumptions are made here.

**Assumption 1:** \( G_0(x) \) and \( G(x) \) are bounded away from zero with the same known sign, for \( x \in \Omega_X \) where \( \Omega_X \) is the desired domain of operation.

**Assumption 2:** There exist finite positive, but not necessarily known constants \( M_F, M_d, M_{F_0}, M_G, M_{\Delta G} < \infty \) and known continuous functions \( \overline{F}(x) \) and \( \overline{G}(x, t) \) such that the following inequalities hold for \( x \in \Omega_X \):
\[
\left| \frac{\Delta_F(x)}{F(x)} \right| \leq M_F, \quad \left| \frac{d(x, t)}{\overline{F}(x)} \right| \leq M_d, \quad \left| \frac{F_0(x)}{\overline{F}(x)} \right| \leq M_{F_0}
\]  
(3)

\( 0 < \frac{G_0(x)}{G(x)} \leq M_G, \quad \left| \frac{\Delta_G(x)}{G(x)} \right| \leq M_{\Delta G} \)  
(4)

where \( \overline{F}(x) \) and \( \overline{G}(x, t) \) are appropriate known functions (bounding functions) which will be used for construction of nonlinear damping terms \([4], [8], [4] \).

**Assumption 3:** The reference trajectory \( y_r(t) \) is appropriately chosen as a sufficiently smooth function such that \( \dot{y}_r \) and \( \ddot{y}_r \) are known and
\[
\mathcal{D}_{y_r} = \{ y_r, \dot{y}_r, \ddot{y}_r \in \Omega_Y \subset \Omega_X, |\dot{y}_r| \leq \overline{y_r}, |\ddot{y}_r| > 0 \}
\]  
(5)

To generate a smooth reference trajectory, we can pass the command trajectory \( y_c \) through a low-pass filter: \( y_r = \frac{y_c}{(\tau s + 1)^3} \).

Our task is to design a theoretically guaranteed high performance controller so that the position \( x_1 \) tracks the reference trajectory \( y_r \) accurately.

III. CONTROLLER DESIGN

In this section, we present the design of the novel robust nonlinear controller with DOB. Stability analysis will be given in the next section. The controller is designed in a backstepping manner \([4] \) as follows.

**Step 1:** Define the position error signal and velocity error signal respectively as
\[
z_1 = x_1 - y_r, \quad z_2 = x_2 - \alpha_1
\]  
(6)

where \( \alpha_1 \) is the virtual input to stabilize \( z_1 \).

Then we have subsystem \( S1 \) as the following.
\[
S1 : \dot{z}_1 = \alpha_1 + z_2 - \dot{y}_r
\]  
(7)

The virtual input \( \alpha_1 \) is designed based on the common PI control technique.
\[
\alpha_1 = -c_{1p}z_1 - c_{1i} \int_0^t z_1 dt + \dot{y}_r
\]  
(8)

where \( c_{1p} > 0, \ c_{1i} > 0 \).

Notice that
\[
\dot{\alpha}_1 = -c_{1p} \dot{z}_1 - c_{1i} z_1 + \ddot{y}_r = -c_{1p}(\dot{x}_2 - \dot{y}_r) - c_{1i} z_1 + \ddot{y}_r
\]  
(9)

Our next step is to stabilize the velocity error \( z_2 \).

**Step 2:** The second subsystem \( S2 \) is obtained as
\[
S2 : \dot{z}_2 = F_0(x) + G_0(x)u - \dot{\alpha}_1 + d(x, t) + \Delta_F(x) + \Delta_G(x)u
\]  
(10)

Taking the error terms as the lumped disturbance \( w \), we have
\[
w = z_2 - (F_0 + G_0(x)u - \dot{\alpha}_1)
\]  
(11a)

\[
= d(x, t) + \Delta_F(x) + \Delta_G(x)u
\]  
(11b)

Therefore, the lumped disturbance \( w \) can be obtained as (11a). However, since \( \dot{z}_2 \) usually amplifies high frequency noise, we have to pass (11a) through a low-pass filter \( Q(s) \) to obtain the estimate of \( w \) as
\[
\hat{w} = Q(s)w
\]  
(12)

This is the so called DOB studied extensively in the literature \([1–3], [5], [6] \). In the paper, we adopt a simple first-order filter
\[
Q(s) = \frac{1}{1 + \lambda s}
\]  
(13)

By compensating the control input by \( \hat{w} \), we have an inner-loop around the controlled plant such that the inner-loop approximates a simple nominal plant model at low-frequencies. And hence a simple controller can be designed for the approximated nominal model. This can be seen as follows. Replacing \( u \) in (10) by \( u = (v - F_0(x) + \dot{\alpha}_1 - \hat{w})/G_0(x) \) and assuming \( \hat{w} \approx w \), we have the following simple nominal model
\[
\dot{z}_2 \approx v
\]  
(14)

where \( v \) is a nominal linear input. This pave the way to design a simple controller. The simplest design is to let \( v = -c_2 z_2 \).

However, it should be pointed that we can only expect \( \hat{w} \approx w = d(x, t) + \Delta_F(x) + \Delta_G(x)u \) at low-frequencies. If the disturbance and model mismatch are fast changing, the estimation error \( w - \hat{w} \) cannot be neglected and even can destroy the stability of the closed-loop in the case of large
model mismatch [8]. Although many efforts have been done to improve the robustness of the DOB based motion controllers [2], [3], [5], [6], the DOB based motion controllers are usually designed according to the linear control theory, even if the actual controlled plant may be strongly nonlinear. Unfortunately, the rigorous stability of these controllers for nonlinear systems has not been well studied in the literature.

To stabilize the subsystem, we design the following controller:

\[ u = u_l + u_w + u_r \]
\[ u_l = \frac{\alpha_{20}}{G_0(x)} \]
\[ u_w = \frac{-\hat{w}}{G_0(x)} \]
\[ u_r = -\sum_{i=1}^{4} u_{d_i} \hat{z}_2 \]

where

\[ \alpha_{20} = -c_2\hat{z}_2 + \hat{\alpha}_1 - F_0(x) \]
\[ u_{d1} = \kappa_{21} \tilde{F}(x) \]
\[ u_{d2} = \kappa_{22} \alpha_{2d} \]
\[ u_{d3} = \kappa_{23} \tilde{d}(x, t) \]
\[ u_{d4} = \kappa_{24} |\hat{w}| \]
\[ \alpha_{2d} = |c_2\hat{z}_2 + \hat{\alpha}_1| + \tilde{F}(x) \]

and \( c_2, \kappa_{21}, \kappa_{22}, \kappa_{23}, \kappa_{24} > 0; \alpha_{20} \) is a feedback controller with nominal model compensation; \( u_w \) is a compensating term by the disturbance observer’s output; \( u_{d1}, u_{d2}, u_{d3}, u_{d4} \) are nonlinear damping terms [4] to counteract \( \Delta_F, \Delta_G \) and \( d(x, t) \) respectively; \( u_{d4} \) is a nonlinear damping term to ensure boundedness of \( \hat{z}_2 \) when \( \hat{w} \) is used.

Applying the designed control input \( u \) to subsystem S22, we have

\[ \dot{z}_2 = -c_2 \hat{z}_2 + G(x) u_r - \hat{w} + \Delta_F(x) + d(x, t) + \Delta_G(x)(u_l + u_w) \]

(17)

If \( w - \hat{w} = 0 \), it is easy to show that \( \hat{z}_2 \) is asymptotically stabilized to zero. However, we can only expect that \( w - \hat{w} \approx 0 \) at low-frequencies. At high-frequencies, we can not neglect the effects of \( w - \hat{w} \). Therefore, it is necessary to investigate if the internal signals of the closed-loop are bounded.

Remark 1: Essentially, the modelling errors and disturbances included in \( w \) are counteracted by \( u_{d1}, u_{d2}, u_{d3}, u_{d4} \). Owing to these nonlinear damping terms, the subsystem is stabilized and \( \hat{z}_2 \) can be made bounded even when the disturbance observer is not used (\( \hat{w} \equiv 0 \)). When the disturbance observer is activated, an extra nonlinear damping term \( u_{d4} \) is required to counteract the side effects of \( \hat{w} \), which will be explained in the stability analysis given in the next section.

IV. STABILITY ANALYSIS

A. Step 1

Applying the virtual input \( \alpha_1 \) to subsystem S1, we have

\[ \dot{z}_1 = z_2 - c_1p z_1 - c_i \int_0^t z_1 dt \]

(18)

In the transfer function form, subsystem S1 can be expressed as

\[ z_1 = \frac{s z_2}{s^2 + c_1 p s + c_1} \]

(19)

Equation (19) can be rewritten into the state-space form:

\[ \dot{z}_1 = A z_1 + B z_2 \]

(20)

where \( z_1 = [\int_0^t z_1 dt, z_1] \)

\[ A = \begin{bmatrix} 0 & 1 \\ -c_{1i} & -1 \end{bmatrix} \]

\[ B = [0, 1]^T \]

(21)

The ISS property of subsystem S1 is described in the following lemma which can be easily proved:

Lemma 1: If the virtual input \( \alpha_1 \) is applied to subsystem S1, and if \( z_2 \) is made uniformly bounded at the next step, then S1 is ISS, i.e., for \( 0 < \lambda_0 < 0, 0 < \rho_0 > 0, \)

\[ |z_1(t)| \leq \lambda_0 e^{-\rho_0 t} |z_1(0)| \]

\[ + \lambda_0 \sup_{0 \leq \tau \leq t} |z_2(\tau)| \]

B. Step 2

Next we show that the boundedness and transient performance of \( z_2 \) can be achieved by the nonlinear damping terms. From equation (17), we have

\[ \frac{d}{dt} \left( \frac{z_2^2}{2} \right) = -c_2 \hat{z}_2 + G(x) u_r z_2 + \Delta_F(x) + \Delta_F(x) z_2 + d(x, t) z_2 \]

\[ + \Delta_G(x)(u_l + u_w) z_2 - \hat{w} \]

\[ = -c_2 \hat{z}_2 - \left[ \frac{c_2}{2} + D_2 \right] z_2 + \frac{\Delta_G(x)}{G_0(x)} \alpha_{20} \]

\[ + \Delta_F(x) z_2 + d(x, t) z_2 - \hat{w} z_2 \]

\[ \leq -c_2 \hat{z}_2 - \left[ \frac{c_2}{2} + D_2 \right] z_2 |z_2| - \mu_2 \]

(22)

where

\[ \mu_2(t) = \frac{\Delta_F(x) + \Delta_G(x) \alpha_{20} + d(x, t) - G(x) \hat{w}}{G_0(x)} \]

\[ \frac{c_2}{2} + D_2 \]

(23)

\[ D_2 = \frac{G(x)}{G_0(x)} \left( \kappa_{21} \tilde{F}(x) + \kappa_{22} \alpha_{2d} + \kappa_{23} \tilde{d}(x, t) + \kappa_{24} |\hat{w}| \right) \]

(24)

According to the inequalities in assumption 2, it is obvious that each term in the numerator of \( \mu_2 \) is counteracted by a nonlinear damping term in the denominator so that \( \mu_2 \) is uniformly bounded. Moreover, from (22) we have

\[ |z_2| \geq \mu_2(t) \Rightarrow \frac{d}{dt} \left( \frac{z_2^2}{2} \right) \leq -c_2 \hat{z}_2^2 \]

(25)

and hence

\[ |z_2(t)| \leq |z_2(0)| e^{-c_2 t/2} + \sup_{0 \leq \tau \leq t} \mu_2(\tau) \]

(26)
Therefore it is easy to understand that the uniform boundedness of $z_2$ can be ensured by the nonlinear damping terms. And hence Lemma 1 implies $|z_{1a}|$ is bounded. Then under Assumption 3 that the reference trajectory $y_r$, $\hat{y}_r$ and $\dot{y}_r$ are uniformly bounded, we can conclude that all the internal signals of the two subsystems are uniformly bounded.

In the above analysis, the main attention is to show the boundedness of the internal signals of the closed-loop. No analysis yet be done for the attenuation effects of $w - \hat{w}$. Without such an analysis, we cannot clearly see how the DOB’s output $\hat{w}$ can bring improvement. We now attempt to make such an effort.

Keeping that all the internal signals are bounded in mind, we rewrite (22) according to the last line of (17).

$$\frac{d}{dt} \left( \frac{z_2^2}{2} \right) = -\frac{c_2}{2} z_2^2 - \left[ \frac{c_2}{2} + D_{2w} \right] z_2^2 + w z_2 - \hat{w} z_2$$

$$\leq -\frac{c_2}{2} z_2^2 - \left[ \frac{c_2}{2} + D_{2w} \right] |z_2| |z_2| - \mu_{2w}$$

where

$$\mu_{2w}(t) = \frac{|w - \hat{w}|}{c_2/2 + D_{2w}}$$

$$D_{2w} = \kappa_{21} F(x) + \kappa_{22} \alpha_{2d} + \kappa_{23} d(x, t) + \kappa_{24} |\hat{w}|$$

Notice that

$$w - \hat{w} = \Delta G(x) G_0(x) (-D_{2w} z_2 + \alpha_{20}) + \Delta F(x)$$

$$+ d(x, t) - \frac{G(x)}{G_0(x)} \hat{w}$$

Provided that all the internal signals are uniformly bounded, it is trivial to verify that each term in the numerator of $\mu_{2w}$ is counteracted by a nonlinear damping term in the denominator. Furthermore, it should be commented here that $\mu_{2w}$ has very transparent physical meaning. At low-frequencies, we can expect $\mu_{2w} \approx 0$. And any nonzero $w - \hat{w}$ at high-frequencies is counteracted by $c_2/2 + D_{2w}$ so that $z_2$ is quite robust against $w - \hat{w}$.

Finally, we have

$$|z_2| \geq \mu_{2w}(t) \Rightarrow \frac{d}{dt} \left( \frac{z_2^2}{2} \right) \leq -\alpha c_2 z_2^2$$

and hence

$$|z_2(t)| \leq |z_2(0)| e^{-\alpha t/2} + \sup_{0 \leq \tau \leq t} \mu_{2w}(\tau)$$

The theoretical results are summarized in the following lemma:

**Lemma 2**: Let Assumptions 1–3 hold. If the control input $u$ is applied to subsystem $S2$, then subsystem $S2$ is ISS and the error signal $z_2(t)$ is uniformly bounded as

$$|z_2(t)| \leq |z_2(0)| e^{-\alpha t/2} + \sup_{0 \leq \tau \leq t} \mu_{2w}(\tau)$$

**Remark 2**: Owing to the nonlinear damping terms, the ISS property described by (26) or (32) still holds when the DOB is not used ($\hat{w} \equiv 0$). See $\mu_2$ in (23) or $\mu_{2w}$ in (28).

### C. ISS property of the overall error system

Lemmas 1 and 2 imply that the overall error system is a cascade of two ISS subsystems. Define the error signal vector

$$z(t) = \left[ f_0^T z_1(t) dt, z_1(t), z_2(t) \right]^T$$

Then along the same lines of the proof of lemma C.4 in [4], we have the following results:

$$|z(t)| \leq \beta_1 e^{-\rho_1 t} |z(0)| + \beta_2 \left[ \sup_{0 \leq \tau \leq t} \mu_{2w}(\tau) \right]$$

where

$$\beta_1 = \sqrt{2} \left( \lambda_0^2 + \frac{\lambda_0^2}{\rho_0} + \frac{\lambda_0}{\rho_0} + 1 \right)$$

$$\rho_1 = \min \left( \frac{\lambda_0}{\rho_0}, \frac{\lambda_0}{\rho_0} + 1 \right)$$

Furthermore, according to (19), we can see that due to the PI controller for the first subsystem, the steady offset component (zero frequency component) of $z_1$ converges to zero. The results are summarized into the following theorem.

**Theorem 1**: Let Assumptions 1–3 hold. All the internal signals are uniformly bounded and the following results hold:

1) The overall error system is ISS such that

$$|z(t)| \leq \beta_1 e^{-\rho_1 t} |z(0)| + \beta_2 \left[ \sup_{0 \leq \tau \leq t} \mu_{2w}(\tau) \right]$$

2) The steady offset of $z_1$ converges to zero.

**Remark 3**: The second conclusion of Theorem 1 does not mean $z_1$ itself converges to zero since $|z_2|$ can only be made small around the origin. However, due to the integral action of PI control, we can make the zero frequency component (steady offset) of $z_1$ converges to zero.

**Remark 4**: According to the results of Lemma 1, Lemma 2 and Theorem 1, the initial error signals $z_1(0)$ and $z_2(0)$ influence the transient performance significantly. To achieve satisfactory transient performance, it is recommended to initialize the reference trajectory $y_r$ appropriately [4]. This can be simply achieved by appropriately initializing the output of the reference filter $y_r = y_r/(\tau s + 1)^3$ as $y_r(0) = x_1(0), \dot{y}_r(0) = x_2(0)$, so that the initial error signals become $z_1(0) = z_2(0) = 0$.

**Remark 5**: The structure of the proposed controller is shown in Fig. 1. Although the robust nonlinear motion controller with DOB is designed in a modern manner, it should be pointed out that the complicated looking controller can be explained as modifications of the conventional PI motion controller with minor-loop, by adding the feedforward term, nonlinear damping term and DOB term to it. Therefore, it is believed that the proposed controller may gain wide acceptance of the engineers of the industrial side.

323
V. APPLICATION TO A MAGNETIC LEVITATION SYSTEM

To demonstrate the efficient performance of the proposed robust nonlinear controller with DOB, extensive simulations and experiments on a one-degree-of-freedom magnetic levitation system have been performed. In this section, we will show how the experimental results reflect the theoretical analysis.

![Diagram of the magnetic levitation system.](image)

**TABLE I**

PARAMETERS OF THE MAGNETIC LEVITATION SYSTEM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>0.54 [kg]</td>
</tr>
<tr>
<td>( g )</td>
<td>9.8 [m/s²]</td>
</tr>
<tr>
<td>( X_\infty )</td>
<td>0.008114 [m]</td>
</tr>
<tr>
<td>( Q )</td>
<td>0.001624 [Hm]</td>
</tr>
</tbody>
</table>

A. Experimental setup

Extensive experiments have been performed on the setup shown in Fig. 2, whose dynamics is governed by

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= F(x) + d(x, t) + G(x)u
\end{align*}
\]

(36)

where

\[
F(x) = g, \ G(x) = G(x_1) = \frac{-Q}{2M(X_\infty + x_1)^2} \quad u = \frac{1}{2} i^2
\]

(37)

\( x_1 \): air gap (vertical position) of the steel ball; \( x_2 \): velocity of the steel ball; \( i \): coil current; \( g \): gravity acceleration; \( M \): mass of the steel ball; \( Q \) and \( X_\infty \): positive constants determined by the characteristics of the coil, magnetic core and steel ball.

The unknown external disturbance \( d(x, t) \) is a sinusoidal signal artificially added to the control input:

\[
d(x_1, t) = 0.5G(x_1) \sin(\pi t)
\]

(38)

Denoting the nominal physical parameters as \( g_0 \), \( M_0 \), \( Q_0 \) and \( X_\infty 0 \), we have the nominal nonlinear functions and modelling errors respectively as the following.

\[
F_0 = g_0 \\
G_0(x_1) = \frac{Q_0}{2M_0(X_\infty 0 + x_1)^2}
\]

(39)

\[
\Delta F = g - g_0 \\
\Delta G(x_1) = G(x_1) - G_0(x_1)
\]

(40)

The physical parameters shown in Table I were identified through closed-loop operational data and are thought to be reliable. The physically allowable operating region of the steel ball shown in Fig. 1 is limited to 0 [m] ≤ \( x_1 \) ≤ 0.013 [m]. The vertical position of the steel ball is measured by a laser distance sensor (KEYENCE LB-60). The levitated steel ball is controlled by a digital control system that consists of a PC with an 1.0-GHz Intel Pentium III Processor which is loaded with Windows 2000 OS, 12 bits A/D and D/A converters, and a current feedback power amplifier. The control algorithm is coded in Borland C++ language and discretized with a sampling interval of 0.004 [s]. The velocity \( x_2 \) is measured by pseudo-differentiation of the measured position \( x_1 \) as \( \Delta x_1 (0.004 s + 1) \).

B. Design of the proposed controller

The following nominal system parameters with large errors were used to verify the robust performance of our controller.

\[
M_0 = 0.30 [kg], \ g_0 = 9.0 [m/s²] \\
X_\infty 0 = 0.0020 [m], \ Q_0 = 0.0015 [Hm]
\]

(41)

The known functions used for nonlinear damping terms (assumption 2) are given as

\[
\overline{F}(x) = g_0, \ \overline{d}(x, t) = |G_0(x_1)|
\]

(42)
The designed controller parameters are shown as follows.

\[
c_{1p} = 40, \quad c_{1i} = 20^2, \quad c_2 = 40 \\
\kappa_{21} = \kappa_{22} = \kappa_{23} = \kappa_{24} = 3 \\
\lambda = 0.02
\]  

(43)

The steel ball was initially at rest with \(x_1(0) = 13\text{[mm]}\) and \(x_2(0) = 0\text{[mm/s]}\), i.e., the steel ball was held on the steel plate shown in Fig. 2 before the controller’s start.

C. Comparative studies with an adaptive robust nonlinear controller

It was verified that in the presence of considerable large errors of the physical parameters and significant disturbance, the control results by the nominal controller with DOB are not acceptable at all. The steel ball may even hit the steel plate under it. However, by our proposed novel controller, very excellent control performance is achieved despite of the large modelling errors and disturbance. This can be seen in Fig. 3. The results reflect the theoretical analysis quite well.

For comparative studies, we also performed the adaptive robust nonlinear controller studied in [7]. The design parameters are omitted due to the limitation of space. The results are shown in Fig. 4. A comparison of Figs. 3 and 4 shows that the proposed new controller is superior over the adaptive robust nonlinear controller. This is mainly due to the fact that the adaptive robust nonlinear controller while being able to cope with uncertain parameters can not compensate unmodelable disturbance \(d(x, t)\) actively.

VI. CONCLUSIONS

In this paper, a novel and theoretically guaranteed robust nonlinear motion controller with DOB for positioning control of a nonlinear SISO mechanical system was proposed. Rigorous stability analysis was performed. The theoretical results were verified through experimental studies. Comparative studies with an adaptive robust nonlinear controller on a magnetic levitation system were carried out as well. It is concluded that at least for the magnetic levitation system under study which has smooth nonlinearity, the proposed novel controller while being simple is superior over the adaptive robust nonlinear controller.

REFERENCES