Decentralized Adaptive Robust Control of Robot Manipulators

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Abstract—In this paper, we propose a decentralized adaptive robust controller for trajectory tracking of robot manipulators. In each local controller, a disturbance observer (DOB) is introduced to compensate the low-passed coupled uncertainties, and an adaptive sliding mode control term is employed to handle the fast-changing components of the uncertainties beyond the passband of the DOB. In contrast to most of the local controllers using DOB for robot manipulators that are based on linear control theory, in this study, by some special nonlinear damping terms, the boundedness of the signals of the overall nonlinear system is first ensured. This paves the way to analyze how the DOB and adaptive sliding mode control play in a cooperative way in each local subsystem to achieve an excellent control performance. Simulation results are included to confirm the theoretical results.

I. INTRODUCTION

Robot manipulators possess many uncertainties with inherent strong nonlinearities. To improve the tracking performance of robot manipulators, significant effort has been made to seek advanced control strategies, such as adaptive control and robust control approaches. However, since most approaches are based on a centralized control structure which requires rather complicated hardware configuration and tedious computation, their practical application may be computationally demanding.

A decentralized control system based only on the local information is highly desirable. However, for manipulator tracking tasks, decentralized approaches are not straightforward since the overall system cannot be decomposed into subsystems such that the states and control inputs are fully decoupled from one another, because of the inherent coupling such as moment of inertia and Coriolis force [1–5].

In this paper, we propose a decentralized adaptive robust controller for trajectory tracking of robot manipulators. In each local controller, a disturbance observer (DOB) is introduced to compensate the low-passed coupled uncertainties, and an adaptive sliding mode control term is employed to handle the fast-changing components of the uncertainties beyond the pass-band of the DOB. In contrast to the DOB based linear local controllers for robot manipulators, in this study, by some special nonlinear damping terms, the boundedness of the signals of the total nonlinear system is first ensured. Differing from some existing works where only the control performance of the total system is analyzed [1–5], we also analyze how the DOB and adaptive sliding mode control play in a cooperative way to achieve an excellent control performance in each local subsystem, provided the boundedness of the total system signals. Simulation results are provided to support the theoretical results.

II. PROBLEM STATEMENT

Consider an $n$—link rigid manipulator governed by the following dynamic model described by the Lagrange-Euler vector equation:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) + f(\dot{\theta}) = u$$

(1)

where $\theta = \theta(t) \in \mathbb{R}^n$ is the joint angle vector; $u \in \mathbb{R}^n$ is the input torque vector; $M(\theta) = M^T(\theta) \in \mathbb{R}^{n \times n}$, $M(\theta) > 0$ is the inertia matrix; $C(\theta, \dot{\theta})\dot{\theta} \in \mathbb{R}^n$ is the centrifugal and Coriolis torque; $g(\theta) \in \mathbb{R}^n$ is the gravity torque; $f(\dot{\theta}) \in \mathbb{R}^n$ is the friction force torque.

The following assumptions are imposed for the system.

**Assumption 1:** All joints of the robot manipulators under consideration are revolute such that Property 1 given later holds.

**Assumption 2:** The reference trajectory $\theta_d(t)$ and the time derivatives $\dot{\theta}_d(t)$ and $\ddot{\theta}_d(t)$ are bounded signals.

The system model (1) has the following properties that will be used in control system analysis and design [6].

**Property 1:** The inertia matrix $M(\theta)$ is symmetric and positive-definite such that

$$\mu_{\text{min}} I \leq M(\theta) \leq \mu_{\text{max}} I$$

(2)

for some constants $\mu_{\text{max}}, \mu_{\text{min}} > 0$.

**Property 2:** The matrix $C(\theta, \dot{\theta})$ satisfies

$$\| C(\theta, \dot{\theta}) \|_2 \leq c_H \| \dot{\theta} \|_2$$

(3)

for some constant $c_H > 0$.

**Property 3:** The gravity vector $g(\theta)$ and the friction force vector $f(\dot{\theta})$ satisfy

$$\| g(\theta) \|_2 \leq c_g, \quad \| f(\dot{\theta}) \|_2 \leq c_{f1} + c_{f2} \| \dot{\theta} \|_2$$

(4)

for some constants $c_g, c_{f1}, c_{f2} > 0$.

**Property 4:** The matrix $[M(\theta)/2 - C(\theta, \dot{\theta})]$ is skew symmetric, i.e.,

$$x^T \left[ \frac{1}{2} M(\theta) - C(\theta, \dot{\theta}) \right] x = 0, \quad \forall x \neq 0$$

(5)

Notice that this property implies that the map $(u - f) \mapsto \dot{\theta}$ is passive.

Our task is to design a decentralized controller where at each joint a local controller using only the local information is constructed, so that $\theta$ tracks its reference $\theta_d$ accurately. To this end, we first define an auxiliary error as

$$r = \dot{e} + \Phi e$$

(6)
where $e = \theta - \theta_0$ is the position-tracking error vector and $\Phi = \text{diag}\{\phi_1, \ldots, \phi_n\} > 0$ is a constant matrix.

Substituting $e$ and $r$ into (1), we have

$$M(\theta)\ddot{r} + C(\theta, \dot{\theta})r = u + \xi$$

(7)

$$\xi = -M(\theta)(\ddot{\theta}_0 - \Phi \dot{e}) - C(\theta, \dot{\theta})(\ddot{\theta}_0 - \Phi \dot{e}) - g(\theta) - f(\dot{\theta})$$

(8)

According to (7), (4) and (8), we have [1], [2], [4], [5]

$$\|\xi\|_2 \leq \alpha_1 + \alpha_2 \|e\|_2 + \alpha_3 \|\dot{e}\|_2 + \alpha_4 \|\dot{\theta}\|_2$$

(9)

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are some positive constants.

For a vector-valued signal $x(t)$, we define a truncated norm for $T > 0$ as $\|x\|_T = \sup_{t \in [0, T]} \|x(t)\|_2$. Then we have [2]

**Lemma 1**: Let Assumptions 1 and 2 hold. If there is a constant $T$ such that $\|r\|_T$ exists, then for all $t \in [0, T]$ we have

$$\|\xi\|_2 \leq \beta_1 + \beta_2 \|r\|_T + \beta_3 \|\dot{r}\|_T^2$$

(10)

for some $\beta_1, \beta_2, \beta_3 > 0$.

**III. CONTROLLER DESIGN**

**A. Introduction of DOB**

Recall the overall system model (7). We introduce here a constant nominal inertia $m_{0i}$ ($i = 1, \ldots, n$) for each joint and lump all the other components of the dynamics as a disturbance term $w_i (i = 1, \ldots, n)$. Then we have

$$M_{0i}\ddot{r}_i = u + w_i$$

(11)

where $M_0 = \text{diag}(m_{01}, \ldots, m_{0n})$ is the nominal inertia matrix, and $w = [w_1, \ldots, w_n]^T$ is the disturbance vector expressed as

$$w = \xi - C(\theta, \dot{\theta})r - (M(\theta) - M_0)\dot{r}$$

(12)

Rewriting each subsystem of (11) as

$$m_{0i}\ddot{r}_i = u_i + w_i$$

(13)

Since calculation of $\dot{r}_i$ by direct differentiation is usually contaminated with high frequency noise, we may pass $w_i$ through a low-pass filter to obtain its estimate as

$$Q_i(s)w_i = Q_i(s)(m_{0i}sr_i - u_i)$$

(14)

This is the so-called DOB studied extensively in the literature [7]. In this a study, we adopt a simple second-order filter

$$Q_i(s) = \frac{1}{(1 + \lambda_i s)^2}$$

(15)

where $\lambda_i > 0$.

However, we can only expect $Q_i(s)w_i \approx w_i$ at low-frequency due to limited pass-band of the DOB. If $w_i$ is fast changing, the estimation error cannot be neglected and hence can even degrade the control performance significantly. Moreover, the DOBs’ outputs $w_i (i = 1, \ldots, n)$ may disturb the other joints mutually. To ease the analysis, a

straightforward and simple idea is to saturate the output of the DOB as

$$\hat{w}_i = \begin{cases} \overline{w}_i & \text{for } Q_i(s)w_i \geq \overline{w}_i \\ Q_i(s)(m_{0i}sr_i - u_i) & \text{for } |Q_i(s)(m_{0i}sr_i - u_i)| < \overline{w}_i \\ -\overline{w}_i & \text{for } Q_i(s)w_i \leq -\overline{w}_i \end{cases}$$

(16)

where $\overline{w}_i > 0$ is a selected upper bound of $|w_i|$. Usually, it is commended to choose a sufficiently large $\overline{w}_i$ so that the saturation action is not active after the transient performance. However, as will be shown by the numerical example, even when $\overline{w}_i$ is not so large such that $\overline{w}_i$ is really saturated and hence the estimation error $(w_i - \hat{w}_i)$ is not sufficiently small, the control performance is still satisfactory, owing to the adaptive sliding mode control term included in the local controller (17) given below. The key point is that the DOB and the adaptive sliding mode control term work in a cooperative manner. That is, if one works more, the other one works less, and vice versa. The advantage of the idea is that we need not a perfect DOB or a perfect sliding mode control. However, owing to their cooperative effects, the problems of high-gain or chattering can be avoided.

**B. Description of the controller**

We design the following local controller using only the local information for each subsystem to ensure the boundedness of the overall system signals and to achieve a satisfactory control performance for each subsystem.

$$u_i = -k_ir_i - \rho_i r_i^3 - \sigma_i |\hat{w}_i|r_i - \hat{\gamma}_i - \frac{\hat{n}_{it}^2}{\eta_{it}}r_i + \delta_i r_i$$

(17)

where $k_i$, $\rho_i$, $\sigma_i$, $\delta_i > 0$ are the control gains, and $\hat{n}_{it}$ is an adaptive parameter (adaptive sliding mode control gain) updated as

$$\dot{\hat{n}}_{it} = \gamma_i (|r_i| - \epsilon_i \hat{n}_{it})$$

(18)

where $\gamma_i \geq 0$ is the adaptive gain and $\epsilon_i \geq 0$ is the leakage parameter that prevents $\hat{n}_{it}$ from growing to be unbounded.

The term $-k_i r_i$ is a simple PD control term. $-\rho_i r_i^3$ is a damping term to suppress the effects of neglected uncertainties of the global system model summarized as $\xi$ in (7) and (8) [2], [4], [5]. $-\hat{\gamma}_i$ is a compensation term by DOB for each local system. Since the DOB’s outputs $\hat{w}_i (i = 1, \ldots, n)$ may disturb mutually through the interconnections of the joints, to suppress the interconnections of $\hat{w}_i (i = 1, \ldots, n)$, we employ the damping term $-\sigma_i |\hat{w}_i|r_i$. Finally, the last term of $u_i$ is a smoothed version of sliding mode control term with an adaptively updated gain $\hat{n}_{it}$. Notice that when $\delta_i \rightarrow 0$, this term approaches a hard switching control action.

The nonlinear damping terms $-\rho_i r_i^3$ and $-\sigma_i |\hat{w}_i|r_i$ play the key roles to ensure the boundedness of the overall system signals, as will be shown later. Intuitively, a larger $|\hat{w}_i|$ or a larger $|\hat{n}_{it}|$ causes a stronger control action. Additionally, the sliding mode control term is adopted to further suppress the estimation error $(w_i - \hat{w}_i)$ beyond the pass-band of DOB.
IV. PERFORMANCE ANALYSIS

We first ensure the boundedness of the overall system signals, based on the properties and assumptions given in the previous section. Then provided the boundedness of the overall system signals, we can analyze the control performance of each local subsystem. Therefore, the performance analysis includes two phases, i.e., analysis of the overall system, and then that of each local subsystem.

A. Analysis of the overall system

The results of analysis are given in Theorem 1. The proof is along the spirit of [2], but with modifications specified by the newly designed controller in this study.

*Theorem 1:* Let Assumptions 1 and 2 hold. For the robot manipulator (1) controlled by the proposed adaptive decentralized robust controller (17), there exists a constant $l_1 > 0$, such that $r$ is bounded as $\|r\| < l_1$ and hence all the internal signals are bounded, provided the following condition.

$$\sqrt{\mu_{\min}} l_1 > \left[ \frac{2n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\rho_{\min}} \right]^{1/3}$$

$$\geq \left( \frac{n \sqrt{\bar{w}_i}}{2 \rho_{\min} \sigma_{\min}} \right)^{1/4}$$

where $\rho_{\min} = \min\{\rho_1, \ldots, \rho_n\} > 0$, and $\sigma_{\min} = \min\{\sigma_1, \ldots, \sigma_n\} > 0$.

*Remark 1:* The first inequality of (19) is easily satisfied for a sufficiently large $l_1$. The second inequality of (19) can be rewritten as

$$\left( \beta_1 + \beta_2 l_1 + \beta_3 l_1^2 \right)^{1/3} \geq 2^{-11/12} n^{1/6} \rho_{\min}^{1/12} \left( \bar{w}_i / \sigma_{\min}\right)^{1/4}$$

Typically, the value of $2^{-11/12} n^{1/6} \rho_{\min}^{1/12}$ is not far away from unity. And for a sufficiently large $l_1$, $\sigma_{\min}$ need not be very large to satisfy the condition if $\bar{w}_i$ is not extremely large.

*Proof*

The proof process includes two steps.

The first step is to show that there exists a constant $l_1 > 0$, such that $r$ is bounded as $\|r\| < l_1$. The conclusion is proved by contradiction. To this end, according to (19) we first let a positive constant $l_1$ satisfy

$$\|r(0)\| < \left[ \frac{2n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\rho_{\min}} \right]^{1/3} \sqrt{\mu_{\min}} l_1 \leq l_1$$

(21)

Notice that the above inequality can be satisfied if $l_1$ is sufficiently large.

Now assume the signal $r(t)$ is not bounded. Thus there always exists a smallest time $T_1$ such that $\|r(T_1)\| = l_1$.

Consider a Lyapunov function candidate according to Property 2

$$V(t, r) = \frac{1}{2} r^T M(\theta) r$$

Taking the derivative along the trajectory of the closed-loop system, and using Property 4, the Schwartz inequality

$$\sum_{i=1}^n r_i^4 \geq \frac{1}{n} \|r\|^2$$

and Lemma 1, we have

$$V(t, r) = r^T \left( M(\theta) r + \frac{1}{2} \dot{M}(\theta) r \right)$$

$$\leq r^T \left( u + \xi(\theta, \dot{\theta}) r + \frac{1}{2} \dot{M}(\theta) r \right)$$

$$\leq r^T \left( u + \|r\| \xi \right)$$

$$= -\sum_{i=1}^n k_i r_i^2 - \sum_{i=1}^n \rho_i r_i^2 - \sum_{i=1}^n \sigma_i |\dot{w}_i| r_i^2 - \sum_{i=1}^n r_i |\dot{w}_i|$$

$$+ \|r\| \xi$$

$$\leq -\rho_{\min} \frac{n}{2n} \|r\|^2 + \sum_{i=1}^n |\dot{w}_i| \left( \sigma_i |r_i| - \frac{1}{2 \sqrt{\sigma_i}} \right)^2$$

$$+ \sum_{i=1}^n \frac{|\dot{w}_i|}{4 \sigma_i} \|r\| \leq -\|r\|^2 \left( \rho_{\min} \|r\|^3 - \beta_1 + \beta_2 l_1 + \beta_3 l_1^2 \right)$$

$$- \left( 2n \sqrt{\bar{w}_i} \right) \|r\|^2 - \frac{n \bar{w}_i}{4 \sigma_{\min}}$$

(23)

By (19), (21) and the assumption that there exists a smallest time $T_1$ such that $\|r(T_1)\| = l_1$, we can say that there exists a time instant $t = T_1 - t_1 > 0$, $t_1 > 0$ such that

$$l_1 > \|r(T_1 - t_1)\| = \left[ \frac{2n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\rho_{\min}} \right]^{1/3}$$

$$\geq \left( \frac{n \sqrt{\bar{w}_i}}{2 \rho_{\min} \sigma_{\min}} \right)^{1/4}$$

(24)

However, according to (23), we have $d/dt V(t) \leq 0$, for all $t \in [T_1 - t_1, T_1]$. Therefore, for all $t \in [T_1 - t_1, T_1]$, we have

$$V(T_1, r(T_1)) \leq V([T_1 - t_1], r(T_1 - t_1))$$

$$\leq \frac{1}{2} \mu_{\max} \left[ \frac{2n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\rho_{\min}} \right]^{2/3}$$

(25)

But the definition of $T_1$ leads to

$$V_1(T_1, r(T_1)) \geq \frac{1}{2} \mu_{\min} l_1^2$$

(26)

Clearly, the last two inequalities are in contradiction, according to (21). This implies that the assumption of $\|r(T_1)\| = l_1$ is false. Thus the error signal vector $r$ is bounded and satisfies $\|r(t)\| < l_1$ for all $t \geq 0$.

The next step is to show that all the internal signals are bounded. Rewrite the adaptive law (18) as the output of a stable low-pass filter driven by $r_i$:

$$\dot{\hat{w}}_i = \frac{\gamma_i |r_i|}{s + \gamma_i \xi_i}$$

(27)

We can conclude that $\hat{w}_i$ is bounded since $r_i$ is bounded. Furthermore, according to Assumption 2, (6) and (9), we conclude that $e, \dot{e}, \theta, \dot{\theta}$ and $\xi$ are bounded. And each local controller $u_i$ is bounded. Therefore, all the internal signals are bounded. □

*Remark 2:* The condition (19) is always satisfied for a sufficiently large bound $l_1$. The results of Theorem 1 only
tell us that the error signal vector \( r \), the adaptive gain vector \( \hat{\eta}_t \) and hence all the internal signals can be made to be bounded. It should be emphasized here that at the present stage our purpose is only to ensure the boundedness of the overall system signals. And hence a conservative bound of the overall error system is acceptable. Later, we will show that each individual error signal \( r_i \) can be made sufficiently small by virtue of the corresponding local controller.

**Remark 3:** Considering the derivation process of the boundedness of \( V(t) \), we can see that the damping terms \(-\rho r_i^2 \) and \(-\sigma_i \hat{\omega}_i |r_i| \) play the key roles to ensure the boundedness of the overall system signals. Even when the smoothed sliding mode control term is removed, the boundedness of the overall system signals is still ensured. The sliding mode control term is introduced to further reduce the control error of each subsystem in the cases where the signals are fast so that the DOB cannot cope with them sufficiently.

**B. Analysis of each subsystem**

Provided the boundedness of the overall system signals, we are ready to analyze how the DOBs and adaptive sliding mode control techniques bring improvement in each subsystem.

Substituting the local controller (17) into the subsystem (13), we have

\[
m_{i0}r_i = -k_ir_i - \rho r_i^3 - \sigma_i \hat{\omega}_i |r_i| - \hat{\omega}_i - \frac{\hat{\eta}_t^2}{\eta_t |r_i| + \delta_i} r_i - w_i \tag{28}
\]

Owing to (16) and Theorem 1, \( w_i(t) \) and \( \hat{\omega}_i(t) \) are bounded. Define

\[
\eta_{t,0}^* = \sup_{0 \leq \tau \leq t} |w_i(\tau) - \hat{\omega}_i(\tau)| \tag{29}
\]

Then we consider the following Lyapunov function.

\[
V_i(t \geq 0, r_i) = \frac{1}{2} m_{i0} r_i^2 + \frac{1}{2} \hat{\eta}_t^2 \tag{30}
\]

where \( \hat{\eta}_t = \hat{\eta}_t - \eta_{t,0}^* \). Taking the derivative along the trajectory of (28), we have

\[
\dot{V}_i(t \geq 0, r_i) = -k_ir_i^2 - \rho r_i^3 - \frac{\hat{\eta}_t^2}{\eta_t |r_i| + \delta_i} r_i^2 - \sigma_i \hat{\omega}_i |r_i|^2 + r_i (w - \hat{\omega}_i) + \hat{\eta}_t (|r_i| - \epsilon_i \hat{\eta}_t) \tag{31}
\]

\[
\leq -k_ir_i^2 - \frac{\hat{\eta}_t^2}{\eta_t |r_i| + \delta_i} r_i^2 + \eta_{t,0}^* |r_i| + (\eta_t - \eta_{t,0}^*) |r_i| - \epsilon_i \hat{\eta}_t \hat{\eta}_t \]

Using the relation

\[
2\eta_t \hat{\eta}_t = \eta_t^2 + \hat{\eta}_t^2 - \eta_{t,0}^* |r_i| \geq \hat{\eta}_t^2 - \eta_{t,0}^* \tag{32}
\]

we have

\[
V_i(t \geq 0, r_i, \hat{\eta}_t) \leq -k_ir_i^2 - \frac{\hat{\eta}_t^2}{\eta_t |r_i| + \delta_i} r_i^2 + \hat{\eta}_t |r_i| + \frac{\epsilon_i}{2} (\eta_{t,0}^* - \hat{\eta}_t^2) \]

\[
\leq -k_ir_i^2 + \delta_i + \frac{\epsilon_i}{2} (\eta_{t,0}^* - \hat{\eta}_t^2) \]

\[
= -2k_i \frac{1}{m_{i0}} \hat{\eta}_t^2 r_i^2 + \eta_{t,0}^* \frac{\hat{\eta}_t^2}{2 |r_i|} + \frac{\epsilon_i}{2} \eta_{t,0}^2 + \delta_i \]

\[
\leq -\zeta_{ci} V_i(t \geq 0, r_i) + \delta_{ci} \tag{33}
\]

where

\[
\zeta_{ci} = \min \left( \frac{2k_i}{m_{i0}}, \epsilon_i \gamma_i \right), \quad \delta_{ci,0} = \frac{\epsilon_i}{2} \eta_{t,0}^2 + \delta_i \tag{34}
\]

And hence we obtain the following transient performance:

\[
V_i(t) \leq V_i(0) e^{-\zeta_{ci} t} + \delta_{ci,0} t \tag{35}
\]

(35) implies that \( V_i(t \geq 0, r_i) \) is bounded by \( \delta_{ci,0}/\zeta_{ci} \) as \( t \to \infty \). In addition, since \( |r_i|^2 \leq 2V_i/m_{i0} \), we have

**Theorem 2:** Let the assumptions and results of Theorem 1 hold. For each subsystem of the decentralized robot manipulator model (13) controlled by the proposed decentralized adaptive robust controller (17), all the internal signals are bounded, and the auxiliary error of each subsystem satisfies

\[
|r_i(t)| \leq \sqrt{\frac{2V_i(0)}{m_{i0}}} e^{-\zeta_{ci} t} + \sqrt{\frac{2\delta_{ci,0}}{m_{i0} \zeta_{ci}}} \tag{36}
\]

Inspection of (34) indicates that \( \delta_{ci,0}/\zeta_{ci} \) can be reduced by choosing sufficiently small values of \( \epsilon_i \) and \( \delta_i \), and a large value of \( \gamma_i \). However, according to (29), \( \eta_{t,0} \) and hence \( \delta_{ci,0} \) may not be small since the initial value \( \hat{\omega}_i(0) \) is usually set to be zero. We then discuss the control performance after a short transient phase of DOB. Let \( t_{si}(\lambda_i) \) be an effective time-constant of the DOB depending on \( \lambda_i \), until which the initial values of \( (w_i - \hat{\omega}_i) \) have decayed out sufficiently such that for a relatively small constant \( \eta_{t,si}^* \), we have

\[
\eta_{t,si}^* = \sup_{t_{si}(\lambda_i) \leq \tau \leq t} |w_i(\tau) - \hat{\omega}_i(\tau)| \tag{37}
\]

and

\[
\delta_{ci,si} = \frac{\epsilon_i}{2} \eta_{t,si}^* + \delta_i \tag{38}
\]

Comparing (29) and (37), it is expected that \( \eta_{t,si}^* \) can be much smaller than \( \eta_{t,0} \). Then by rewriting (36), we have

**Corollary 1:** Let the assumptions and results of Theorem 1 hold. The auxiliary error of each subsystem satisfies

\[
|r_i(t)| \leq \sqrt{\frac{2V_i(t_{si})}{m_{i0}}} e^{-\zeta_{ci} (t - t_{si})} + \sqrt{2\delta_{ci,si} / m_{i0} \zeta_{ci}} \tag{39}
\]

The results of Corollary 1 imply that a sufficiently small ultimate error \( \sqrt{2\delta_{ci,si}/(m_{i0} \zeta_{ci})} \) can be achieved by the cooperative effects of the DOB and the adaptive sliding mode control. However, since the Lyapunov function \( V_i(t) \) includes also \( \hat{\eta}_t \), i.e., the error term of the adaptive parameter,
\[ h_1(\theta, \dot{\theta}) = -m_2 l_2(\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) \sin(\theta_2) \]
\[ h_2(\theta, \dot{\theta}) = m_3 l_2 \dot{\theta}_2^2 \sin(\theta_2) \]
\[ g_1(\theta) = g(m_1 l_1 + m_2 l_1 \cos(\theta_1) + m_2 l_2 \cos(\theta_1 + \theta_2) \]
\[ g_2(\theta) = m_2 g l_2 \cos(\theta_1 + \theta_2) \]
\[ f_1(\theta) = 0 \]
\[ f_2(\theta) = 0 \]  

where the physical parameters are given as follows. \( g = 9.807 \text{[m/s}^2] \), \( m_1 = 4.0 \text{[kg]} \), \( m_2 = 2.0 \text{[kg]} \), \( l_1 = 0.5 \text{[m]} \), \( l_2 = 0.25 \text{[m]} \), \( l_{c1} = 0.25 \text{[m]} \), \( l_{c2} = 0.15 \text{[m]} \), \( I_1 = 1.0 \text{[kg-m}^2] \), \( I_2 = 0.8 \text{[kg-m}^2] \).

We here consider the following reference trajectories.
\[ \theta_{d1}(t) = 0.2 + 2.0 \sin(4.0t) \]
\[ \theta_{d2}(t) = -1.7 + 1.8 \cos(2.0t) \]  

The design parameters are given as follows.
\[ \phi_1 = \phi_2 = 10 \]
\[ \rho_1 = \rho_2 = 10 \]
\[ \lambda_1 = \lambda_2 = 0.03 \]
\[ \gamma_1 = \gamma_2 = 0.05 \]
\[ \alpha_1 = \alpha_2 = 5 \]  

The initial adaptive parameters are set as \( \hat{\eta}_{10} = \hat{\eta}_{20} = 0 \). Additionally, the saturation levels of the DOBs are chosen as \( \hat{w}_1 = \hat{w}_2 = 150 \) (see (16)).

The controller is implemented at a sampling period of \( T = 0.2 \text{[ms]} \). The measurements of \( \dot{\theta}_1 \) and \( \dot{\theta}_2 \) are obtained by pseudodifferentiations of the position measurements due to \( s/(0.001 s + 1) \).

The first simulation results are shown in Fig. 1, where from the top to the bottom are respectively the position-tracking errors \( e_1 \), \( e_2 \), auxiliary errors \( r_1 \), \( r_2 \), control signals \( u_1 \), \( u_2 \), DOBs' outputs \( \hat{w}_1 \), \( \hat{w}_2 \), adaptive parameters \( \eta_{1t} \), \( \eta_{2t} \). It can be found in Fig. 1 that the proposed controller delivers a very excellent position-tracking performance.
As suggested in (16), the DOB outputs are saturated to ease the performance analysis. When \( \overline{w}_1 \) is not so large and hence \( \tilde{w}_1 \) is really saturated, the estimation error \( (w_i - \tilde{w}_i) \) may not be sufficiently small. In this case, however, owing to the adaptive sliding mode control terms, the control performance may not degrade significantly. To confirm this, we repeat the simulation of Fig. 1 under the same conditions except that the saturation levels are reduced to \( \overline{w}_1 = \overline{w}_2 = 70 \). The results are shown in Fig. 2. It can be verified that although \( \tilde{w}_1 \) hits the saturation level, the control results of Fig. 2 are still similar to those in Fig. 1.

Finally, for the same controller used for the results of Fig. 1, we investigate the control performance when the values of the physical parameters \( m_1, m_2, I_1, I_2 \) changed to be 150\% of their original values at the time instant of 10[sec], i.e., That is, \( m_1 = 4.0 \rightarrow 6.0[\text{kg}], m_2 = 2.0 \rightarrow 3.0[\text{kg}], I_1 = 1.0 \rightarrow 1.5[\text{kgm}^2], I_2 = 0.8 \rightarrow 1.2[\text{kgm}^2] \) at 10[sec]. The results are shown in Fig. 3. It can be verified that due to the cooperative effects of nonlinear damping terms, DOB, and adaptive sliding mode control term, the control performance is not sensitive. However, it can be observed the amplitudes of the control signals and their DOBs’ outputs after 10[sec] are quite different from those before 10[sec].

Finally, we should mention here that we have verified that the position-tracking performance outperforms those of [2], [5] significantly, for the same physical parameters, reference trajectories and initial conditions. The results are however not shown here due to the limitation of space.

VI. CONCLUSIONS

In this study, a decentralized adaptive robust controller for trajectory tracking of robot manipulators has been proposed. For manipulator tracking tasks, design of a decentralized controller is not straightforward since all of the subsystems are strongly interconnected. To tackle these interconnections, at each subsystem, a DOB is introduced to compensate the low-passed coupled uncertainties, and an adaptive sliding mode control term is employed to handle the fast-changing components of the uncertainties beyond the pass-band of the DOB. By some special nonlinear damping terms, the boundedness of the overall system signals is ensured. Furthermore, we have analyzed how the DOB and adaptive sliding mode control play in a cooperative way in each local subsystem to achieve an excellent control performance, i.e., the control performance of each subsystem has been analyzed rigorously. The theoretical claims have been confirmed by the simulation results. Compared to most centralized adaptive control methods for robot manipulators, the proposed controller while delivering a very excellent control performance, is considered to be very simple and requires moderate computational burden. This works is partly supported by JSPS (Grant number 23560520).

REFERENCES